

Problem 1.13

[Difficulty: 5]

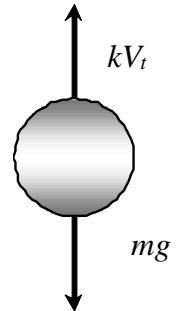
1.13 For Problem 1.12, find the distance the particles travel before reaching 99 percent of terminal speed. Plot the distance traveled as a function of time.

Given: Data on sphere and terminal speed from Problem 1.12.

Find: Distance traveled to reach 99% of terminal speed; plot of distance versus time.

Solution: Use given data; integrate equation of motion by separating variables.

The data provided are: $M = 1 \times 10^{-13} \cdot \text{slug}$ $V_t = 0.2 \cdot \frac{\text{ft}}{\text{s}}$



Newton's 2nd law for the general motion is (ignoring buoyancy effects) $M \cdot \frac{dV}{dt} = M \cdot g - k \cdot V$ (1)

Newton's 2nd law for the steady state motion becomes (ignoring buoyancy effects) $M \cdot g = k \cdot V_t$ so $k = \frac{M \cdot g}{V_t}$

$$k = 1 \times 10^{-13} \cdot \text{slug} \times 32.2 \cdot \frac{\text{ft}}{\text{s}^2} \times \frac{\text{s}}{0.2 \cdot \text{ft}} \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}} \quad k = 1.61 \times 10^{-11} \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}}$$

To find the distance to reach 99% of V_t , we need $V(y)$. From 1: $M \cdot \frac{dV}{dt} = M \cdot \frac{dy}{dt} \cdot \frac{dV}{dy} = M \cdot V \cdot \frac{dV}{dy} = M \cdot g - k \cdot V$

Separating variables $\frac{V \cdot dV}{g - \frac{k}{M} \cdot V} = dy$

Integrating and using limits $y = -\frac{M^2 \cdot g}{k^2} \cdot \ln\left(1 - \frac{k}{M \cdot g} \cdot V\right) - \frac{M}{k} \cdot V$

We must evaluate this when $V = 0.99 \cdot V_t$ $V = 0.198 \cdot \frac{\text{ft}}{\text{s}}$

$$y = \left(1 \cdot 10^{-13} \cdot \text{slug}\right)^2 \cdot \frac{32.2 \cdot \text{ft}}{\text{s}^2} \cdot \left(\frac{\text{ft}}{1.61 \cdot 10^{-11} \cdot \text{lbf} \cdot \text{s}}\right)^2 \cdot \left(\frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}}\right)^2 \cdot \ln\left(1 - 1.61 \cdot 10^{-11} \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}} \cdot \frac{1}{1 \cdot 10^{-13} \cdot \text{slug}} \cdot \frac{\text{s}^2}{32.2 \cdot \text{ft}} \cdot \frac{0.198 \cdot \text{ft}}{\text{s}} \cdot \frac{\text{slug} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2}\right) \dots$$

$$+ 1 \cdot 10^{-13} \cdot \text{slug} \times \frac{\text{ft}}{1.61 \cdot 10^{-11} \cdot \text{lbf} \cdot \text{s}} \times \frac{0.198 \cdot \text{ft}}{\text{s}} \times \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}}$$

$$y = 4.49 \times 10^{-3} \cdot \text{ft}$$

Alternatively we could use the approach of Problem 1.12 and first find the time to reach terminal speed, and use this time in $y(t)$ to find the above value of y :

From 1, separating variables $\frac{dV}{g - \frac{k}{M} \cdot V} = dt$

Integrating and using limits $t = -\frac{M}{k} \cdot \ln\left(1 - \frac{k}{M \cdot g} \cdot V\right)$ (2)

We must evaluate this when

$$V = 0.99 \cdot V_t \quad V = 0.198 \cdot \frac{\text{ft}}{\text{s}}$$

$$t = 1 \times 10^{-13} \cdot \text{slug} \times \frac{\text{ft}}{1.61 \times 10^{-11} \cdot \text{lbf} \cdot \text{s}} \cdot \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}} \cdot \ln \left(1 - 1.61 \times 10^{-11} \cdot \frac{\text{lbf} \cdot \text{s}}{\text{ft}} \times \frac{1}{1 \times 10^{-13} \cdot \text{slug}} \times \frac{\text{s}^2}{32.2 \cdot \text{ft}} \times \frac{0.198 \cdot \text{ft}}{\text{s}} \times \frac{\text{slug} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2} \right)$$

$$t = 0.0286 \text{ s}$$

From 2, after rearranging

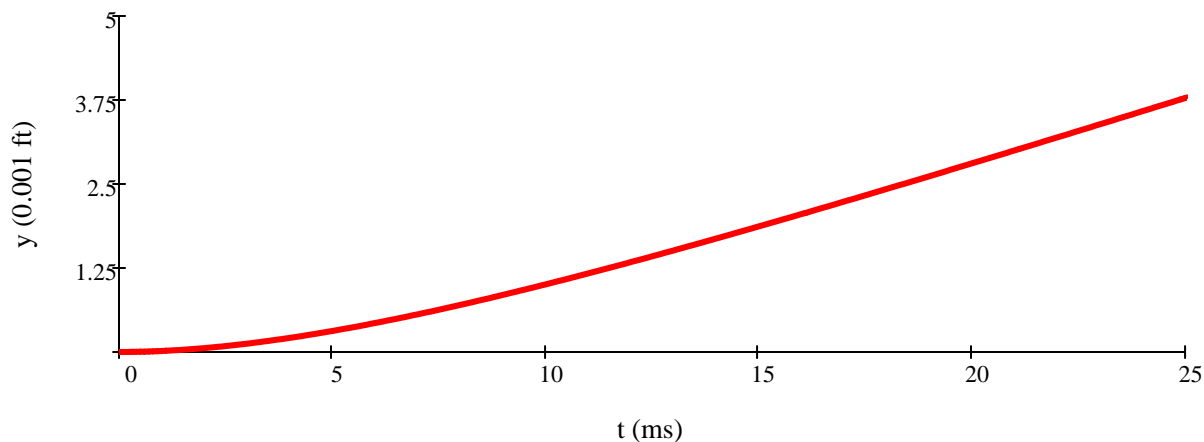
$$V = \frac{dy}{dt} = \frac{M \cdot g}{k} \cdot \left(1 - e^{-\frac{k}{M} \cdot t} \right)$$

Integrating and using limits

$$y = \frac{M \cdot g}{k} \cdot \left[t + \frac{M}{k} \cdot \left(e^{-\frac{k}{M} \cdot t} - 1 \right) \right]$$

$$y = 1 \times 10^{-13} \cdot \text{slug} \times \frac{32.2 \cdot \text{ft}}{\text{s}^2} \times \frac{\text{ft}}{1.61 \times 10^{-11} \cdot \text{lbf} \cdot \text{s}} \cdot \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}} \cdot \left[0.0291 \cdot \text{s} \dots \right. \\ \left. + 10^{-13} \cdot \text{slug} \cdot \frac{\text{ft}}{1.61 \times 10^{-11} \cdot \text{lbf} \cdot \text{s}} \cdot \frac{\text{lbf} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}} \cdot \left(e^{-\frac{1.61 \times 10^{-11}}{1 \times 10^{-13}} \cdot 0.0291} - 1 \right) \right]$$

$$y = 4.49 \times 10^{-3} \cdot \text{ft}$$



This plot can also be presented in Excel.